AP Calculus BC Summer Assignment

Dear AP Calculus BC Student,

You are currently registered for Advanced Placement Calculus BC next fall and I look forward to working with you. Students have been successful in the past because of their dedication and **effort**.

To aid in the your success, a packet has been created containing several exercises of prerequisite skills. During the summer break, please work all the problems ***without*** a calculator. At some point between now and the start of school this fall, you are expected to ***complete all exercises*** either in the packet or on separate sheets of paper. You must show all work and it must be written mathematically correct. ***This packet is due the first day of school.***

Note: If you have not done the AP Calculus AB Summer Assignment, you will need to do that one too!

Thank you in advance for your time this summer to make next year easier and more productive for all involved. We should have a great year, and I am enthusiastic to begin working with all of you.

Have a great summer!

Dr. Browdy

Calculus BC Summer Assignment

Name: ____ Date: _____________

1. Locate the absolute extrema of the given function on the closed interval [–6,6].

$$f(x) = \frac{6x}{x^2+9}$$

2. Find all values of x (if any) at which the graph of the function $-5x+4e^x$ has a horizontal tangent.

3. Find constants a and b such that the function

$$f(x) = \begin{cases} 2, & x \le -3 \\ ax+b, & -3 < x < 1 \\ -2, & x \ge 1 \end{cases}$$

is continuous on the entire real line.

4. Suppose that $\lim_{x \to c} f(x) = -4$ and $\lim_{x \to c} g(x) = 8$. Find the following limit:
$\lim_{x \to c} \left[f(x)g(x) \right]$

5. Find $f'(t)$ if $f(t) = t^5 10^{4t}$.

6. Determine whether Rolle´s Theorem can be applied to the function $f(x) = x^2 - 10x + 16$ on the closed interval [2,8]. If Rolle´s Theorem can be applied, find all values of c in the open interval (2,8) such that $f'(c) = 0$.

7. Find the derivative of the function.

$$P(v) = 30v^5 + 4\sec(v)$$

8. Find the limit.

$$\lim_{x\to\infty}\left(-\frac{5}{6}x-\frac{6}{x^3}\right)$$

9. Determine whether Rolle´s Theorem can be applied to the function $f(x)=\dfrac{(x-16)(x+14)}{(x+17)^2}$ on the closed interval $[-14,16]$. If Rolle´s Theorem can be applied, find all numbers c in the open interval $(-14,16)$ such that $f'(c)=0$.

10. Find the slope of the graph of the function at the given value.

$f(x)=-3x^3-4x^2$ when $x=5$

11. Find the x-values (if any) at which the function $f(x)=\dfrac{x}{x^2-100}$ is not continuous. Which of the discontinuities are removable?

12. Determine whether the Mean Value Theorem can be applied to the function $f(x)=x^3$ on the closed interval [0,8]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (0,8) such that $f'(c)=\dfrac{f(8)-f(0)}{8-0}$.

13. Find an equation of the tangent line to the graph of $y=10^{-x}$ at the point $(-1,10)$.

14. Determine the open intervals on which the function $f\left(x\right)=7xe^{3x}$ is increasing.

15. Use the product rule to differentiate.

$g(x)=x^{-3}\cos\ x$

16. Find the derivative of the function $y=\ln(\ln x^{45})$.

17. Let $f(x) = x^2 + 5$ and $g(x) = 2x$. Find the limits:

(a) $\lim_{x \to 5} f(x)$ (b) $\lim_{x \to 5} g(x)$ (c) $\lim_{x \to -1} g(f(x))$

18. Find the limit:

$$\lim_{x \to \frac{\pi}{3}} \sin x$$

19. Find the limit.

$$\lim_{x \to \infty} \frac{x - 5}{5x - 4}$$

20. Find the derivative of the function.

$$y = \cos\left(4x^5 + 5\right)$$

21. Find an equation of the line that is tangent to the graph of *f and* parallel to the given line.

$$f(x) = 6x^2, \quad -24x - y + 4 = 0$$

22. Find the limit:

$$\lim_{x \to -6} \frac{x^2 + 6x}{\left(x^2 + 36\right)\left(x + 6\right)}$$

23. Find the limit.

$$\lim_{x \to \infty} \frac{-7x + 4}{\sqrt{4x^2 - 4x}}$$

24. Determine the open intervals on which the graph of $y = -8x^3 + 5x^2 + 6x - 3$ is concave downward or concave upward.

25. For the function $f(x) = 2x^3 - 15x^2 + 2$:

(a) Find the critical numbers of *f* (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

Then use a graphing utility to confirm your results.

26. Find the derivative of the function.

$$f(x) = -5x^3 - 5\sin(x)$$

27. Determine the limit (if it exists):

$$\lim_{x \to 0} \frac{\sin x\left(1 - \cos x\right)}{2x^8}$$

28. Determine the open intervals on which the graph of $f(x) = x - 8\cos x$ is concave downward or concave upward.

29. Differentiate the function $f\left(x\right) = \ln\left(7x^2 + 3x + 3\right)$.

30. Find the absolute maximum of the function $f\left(x\right) = x^2 e^{-8x}$ on the closed interval $\left[\frac{1}{8}, \frac{1}{2}\right]$.

31. Find the second derivative of the function.

$$f(x) = \sin 2x^4$$

32. Determine all values of *x*, (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = x^3 + 6x^2 + 5$$

33. Find the derivative of the algebraic function.

$$g(t) = \left(t^3 + 5\right)^6$$

34. Find the derivative of the function.

$$f(x) = x^9$$

35. Find d^2y/dx^2 in terms of x and y.

$$2 - 10xy = 9x - 5y$$

36. Find the slope of the tangent line to the graph of the function at the given point.

$$f(x) = -2x^2 + 3, \quad (-3, -15)$$

37. Determine the open intervals on which the function $f\left(x\right) = 5xe^{8x}$ is decreasing.

38. Find the points of inflection and discuss the concavity of the function $f(x) = -6x + 4\cos x$ on the interval $[0, 2\pi]$.

39. Use implicit differentiation to find $\dfrac{dy}{dx}$.

$$x^5 + 7\ln y = 8$$

40. Use the quotient rule to differentiate.

$$g(t) = \frac{\sin t}{t^2 + 6}$$

41. Find the derivative of the function.

$$f(x) = x^3\sqrt{6 - 7x}$$

42. Find the derivative of the function $f\left(x\right)=\dfrac{e^x+3}{e^x-3}$. Simplify your answer.

43. Find the x-values (if any) at which the function $f(x)=\dfrac{x-7}{x^2-11x+28}$ is not continuous. Which of the discontinuities are removable?

44. For the function $f(x)=(x-1)^{2/3}$:

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

Use a graphing utility to confirm your results.

45. Find the points of inflection and discuss the concavity of the function $f(x)=x\sqrt{x+8}$.

46. Find *dy/dx* by implicit differentiation.

$$x^6+4x+5xy-y^3=25$$

47. Find the derivative of the function.

$$f(t)=(4+8t)^{\frac{5}{9}}$$

Limits and Continuity

1. $\lim_{x\to\infty} \frac{x^2+3x-7}{x^2+x^3} =$

2. $\lim_{x\to a} \frac{\cos x - \cos a}{x-a} =$

3. $\lim_{x\to\infty} \frac{2x^3+2x^2-8x}{3x^3+4x} =$

4. $\lim_{x\to 0} \frac{x}{\sin 5x} =$

5. $\lim_{x\to 0} \frac{1-\cos x}{x} =$

6. $\lim_{x\to -\infty} \frac{x^3+2x-5}{x-6} =$

7. $\lim_{x\to 0} \frac{(2+x)^3-8}{x} =$

8. Find a value of *k* that makes $f(x) = \begin{cases} x^2, & x \le 1 \\ \sin(kx), & x > 1 \end{cases}$ continuous at $x = 1$.

9. If a function f is discontinuous at x = 4, which of the following must be true?
   - I. $\lim_{x\to 4} f(x)$ does not exist
   - II. $f(4)$ does not exist
   - III. $\lim_{x\to 4-} f(x) \ne \lim_{x\to 4+} f(x)$
   - IV. $\lim_{x\to 4} f(x) \ne f(4)$

10. For each part draw an example of a function that satisfies the conditions:
    - a. $f(3)$ exists but $\lim_{x\to 3} f(x)$ does not exist.
    - b. $\lim_{x\to 3} f(x)$ exists but $f(3)$ does not exist.

Derivatives

For exercises 11-15, find the derivative of the given function

11. $y = \dfrac{2x+1}{2x-1}$

12. $y = x^5 - \dfrac{1}{8}x^2 + \dfrac{1}{4}x$

13. $r = \ln\left(\cos^{-1} x\right)$

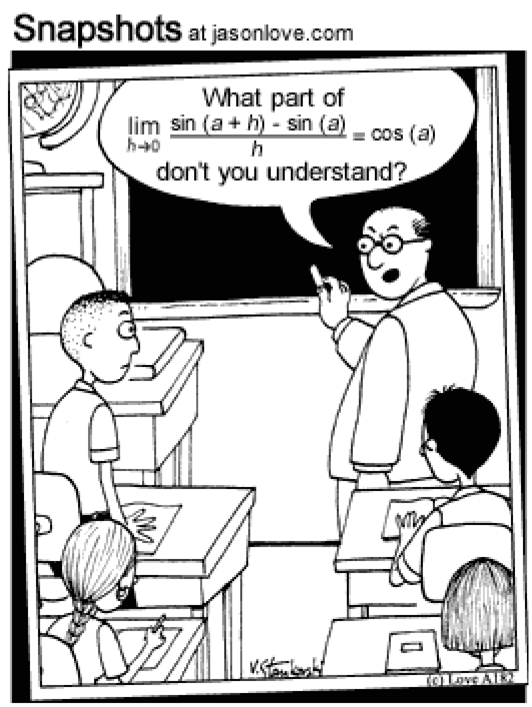
14. $y = x\sqrt{2x+1}$

15. $s = \cos(1-t)$

16. Find $\dfrac{dy}{dx}$ for $xy + 2x^3 + 3y^2 = 1$

17. Find the equation of the tangent line to $y = \sqrt{x^2 - 2x}$ at $x = 3$

18. Consider the function $f(x) = x^3 + 3x^2 - 9x + 3$. On what intervals is this function:
    a. Increasing
    b. Decreasing
    c. Concave Up
    d. Concave Down
    e. What are the local extrema for this function:
    f. What are the inflection points for this function?



19. Let *f* be a function with $f'(x)=\sqrt{x^2+3}$ and $f(1)=6$
   a. Write the linear approximation (tangent line) of *f* at x = 1
   b. Using your equation from part a, approximate $f(1.1)$
   c. Is part b is an under-approximation or over-approximation? (Hint: use *f''*)

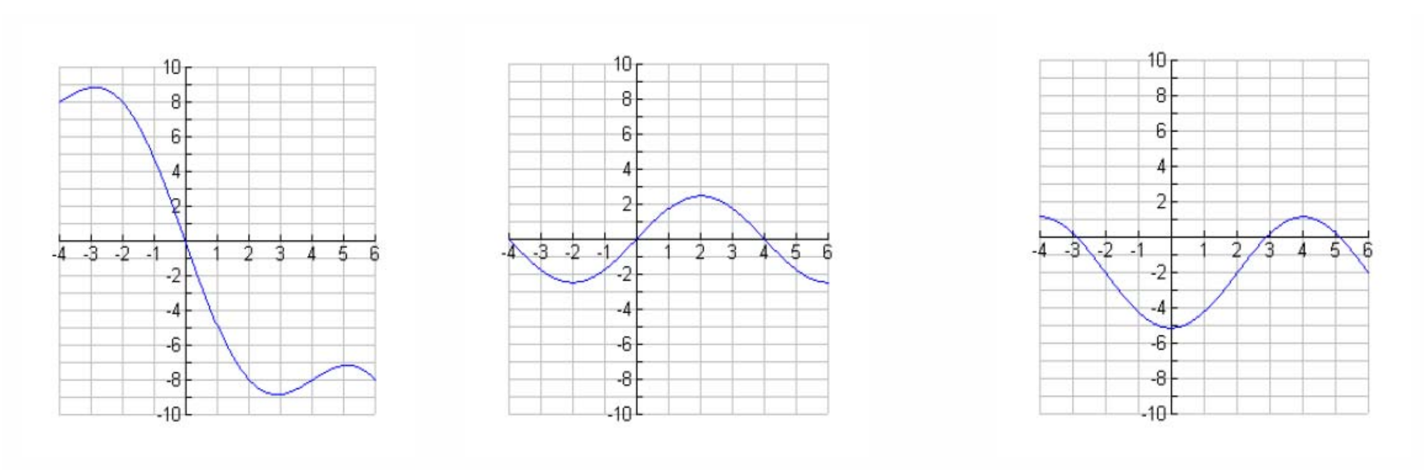
20. Suppose that $f'(2)=4$, $g'(2)=-3$, $f(2)=-1$, $g(2)=1$, $f'(1)=2$, and $g'(-1)=5$. Find the derivatives at x = 2 of the following functions:
   a. $h(x)=f(g(x))$

   b. $k(x)=g(f(x))$

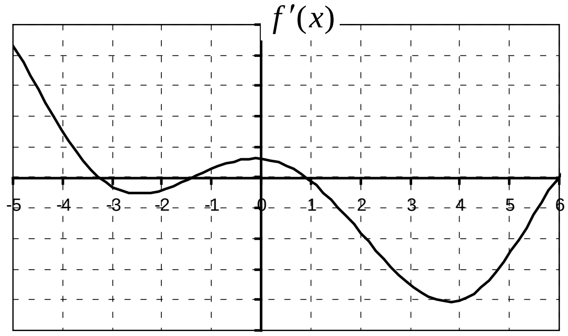
   c. $r(x)=f(x)g(x)$

   d. $s(x)=\dfrac{f(x)}{g(x)}$

21. The graphs (i), (ii) and (iii) given in the figure below are the graphs of a function *f*, and its first two derivatives, *f'* and *f''*, though not necessarily n that order. Identify which of these graphs is the graph of *f*, which is *f'*, and which is *f''*. Justify your answer.



22. The graph of $f'(x)$ is shown below.



a. On what interval(s) is $f(x)$ increasing (approximately)? Decreasing? Explain.

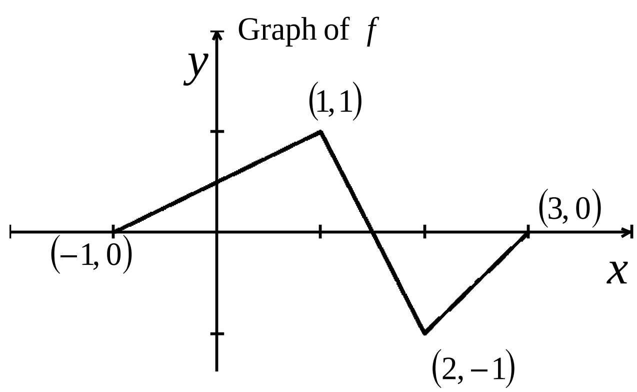
b. Find the x- values of all critical points of $f(x)$ and determine whether they are a maximum, minimum, or neither.

c. On what interval(s) is $f(x)$ concave up? Concave down? Explain.

d. Find the x-values of the inflection point(s) of $f(x)$.

23. If the figure shows the graph of *f*, and $g(x)=\int_0^x f(t)dt$, find:

a. $g(0), g(1), g(2)$, and $g(3)$



b. $g'(0), g'(1), g'(2)$, and $g'(3)$

c. the interval(s) where *g* is increasing

d. the interval(s) where *g* is concave up

Integrals

For exercises 24-28, evaluate the integral.

24. $\int_{-2}^{2} 5dx$

25. $\int_{0}^{\frac{\pi}{3}} \sec^2 \theta d\theta$

26. $\int_{0}^{1} \frac{36}{(2x+1)^3} dx$

27. $\int_{0}^{2} \sqrt{4-x^2}\, dx$ (Hint: interpret this as an area and use Geometry)

28. $\frac{d}{dx} \int_{2}^{x^2} \sqrt{2+\cos^3 t}\, dt$

29. Compute the T_4 (trapezoidal with 4 subintervals) approximation for $\int_{0}^{2} (4x - x^2) dx$ using subintervals of equal length. (Set-up is sufficient – you do not need to simplify your answer.)

*Note: You may use a calculator to evaluate the integrals you use on this page, but please show **all** set-up!*

30. Find the area enclosed between $y = 4 - 2x$ and $y = 4 - x^2$

31. The base of a solid is the region enclosed between the graphs of $y = \sin x$ and $y = -\sin x$ from $x = 0$ to $x = \pi$. Each cross section perpendicular to the x-axis is a semicircle with diameter connecting the two functions. Find the volume of the solid.

32. Find the volume of the solid generated by revolving the region bounded by the following functions about the x-axis: $y = x^2$, $y = 0$, $x = 2$

33. Find the volume of the solid generated by revolving the region from #32 around the line $y = -2$

Review – Polar Equations

Convert the following polar coordinates to rectangular coordinates:

34. (i) $\left(4, \frac{\pi}{2}\right)$ (ii) $(-3, \pi)$

Convert the following rectangular coordinates to polar coordinates:

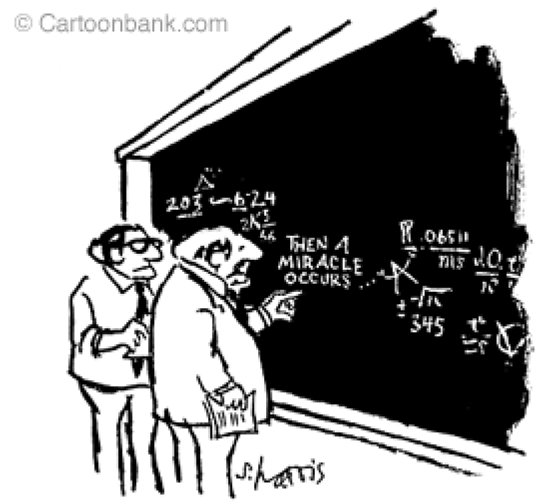
35. (i) $(0, -5)$ (ii) $(1, \sqrt{3})$

Review –Summation Notation

Expand the following:

36. $\sum_{n=0}^{4} \frac{n^2}{2}$

37. $\sum_{n=1}^{3} \frac{1}{n^3}$



"I think you should be more explicit here in step two."

Congratulations! You finished the Summer packet and are ready to start AP Calculus BC!!!